Fault Tolerant Threshold Bounds
or
Rules to be Broken

QuaCGR Talk

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Overview

Question
How does the error model affect the threshold?

Impasse
Generically determining the threshold is difficult.

Observation
Thresholds are relatively insensitive to many details of the correction procedure.

Approach
Bound the threshold using a best-case correction procedure. Consider how the error model affects the bound.

Ground Rules

1. Data is encoded using many qubits in a CSS code.

2. ${}^{C}Z$ is the only two qubit gate.

3. ${}^{C}Z$ gates are sometimes applied between logical data blocks.

4. Perfect X and Z error correction are desired.

5. The recovery operator is determined using information obtained from the operations on the data block being corrected.

The Ultimate Error Correction Protocol

Define a superior error correction protocol.

Given my restrictions, no hypothetical protocol could do better than one in which

1. All ancillae are perfect.

2. The best possible recovery is chosen given only knowledge of the location of errors measured on the ancillae.

3. All recovery unitaries are performed perfectly.

Results

Let α and β index the probabilities of one and two qubit gate errors respectively. $\alpha \in \{X, Y, Z\}$
$\beta \in \{IX, IY, IZ, XI, XY, XZ, YI, YX, YY, YZ, ZI, ZY, ZZ\}$

Model Description	Gate Error Probabilities	Threshold Bound
Modified Reichardt	$p_\alpha = 0$ $p_\beta = \frac{p}{16}$	$p < .033$
Full Depolarizing	$p_\alpha = \frac{p}{3}$ $p_\beta = \frac{p}{15}$	$p < .021$
Two Qubit Depolarizing	$p_\alpha = 0$ $p_\beta = \frac{p}{15}$	$p < .031$
Only IX errors	$p_\alpha = 0$ $p_{IX} = p$ $p_{\beta \neq IX} = 0$	$p < .046$

Rules to Break

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5. The recovery operator is determined using information obtained from the operations on the data block being corrected.

References

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